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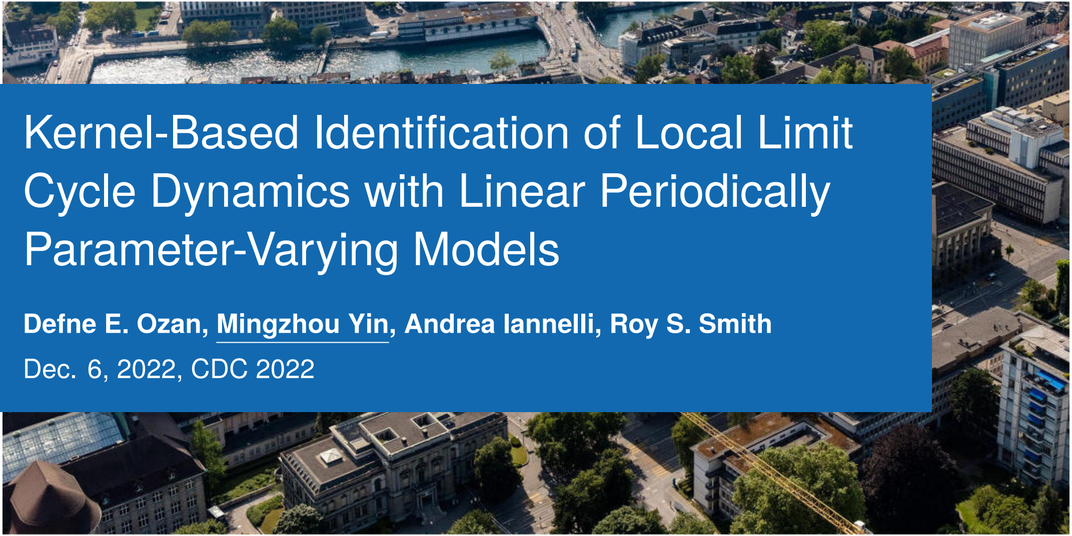

Kernel-Based Identification of Local Limit Cycle Dynamics with Linear Periodically Parameter-Varying Models

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Dec. 6, 2022, CDC 2022

Identification for closed-loop performance analysis

- Control performance deteriorate due to model mismatch
- High fidelity closed-loop model desired for analysis
- ... hard to obtain from first-principle or purely black-box identification

Additional knowledge:

- Reference trajectory of the closed loop (often periodic)
- Closed-loop trajectory is converging to reference

Identification for closed-loop performance analysis

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This work: Identify closed-loop dynamics by

1. modeling as **linear periodic system** using known reference trajectory
2. learning model parameter function by **kernel-based identification** from data

Modeling: making use of the limit cycle

- When controlled along a periodic trajectory
- Closed loop exhibits **limit cycle** behavior

- If reference trajectory $\{x^{\star}(\tau) \mid \tau \in [0, T)\}$ is known
- $\rightarrow$ Modelled as periodic systems w.r.t. orbit location

- If interested in dynamics close to reference
- $\rightarrow$ **Local linear periodic model**

Transverse dynamics projection

- Define transversal hyperplanes around orbit

$$\{S(\tau) \,|\, \dot{x}^\star(\tau) \notin S(\tau), \tau \in [0, T)\}$$

- Map states to **transverse coordinates**

$$x \to (x_\perp, \tau), \quad x_\perp : \text{coordinates on } S(\tau)$$

$\dot{x}_\perp$: converging dynamics to orbit
$\dot{\tau}$: speed along the orbit

- Linearization at the orbit: $x_\perp = 0$

$$\dot{x} = f(x)$$

$\Downarrow$ proj.

$$\begin{cases} \dot{x}_\perp &= f_\perp(x_\perp, \tau) \\ \dot{\tau} &= f_\tau(x_\perp, \tau) \end{cases}$$

$\Downarrow$ approx.

$$\begin{cases} \dot{x}_\perp &= A(\tau) x_\perp \\ \dot{\tau} &= 1 + g(\tau) x_\perp \end{cases}$$

Identification: a function learning problem

- State & state derivative data: $x(t_k), \dot{x}(t_k)$
- Projection onto transverse coordinates[1]: $x_\perp(t_k), \tau(t_k), \dot{x}_\perp(t_k), \dot{\tau}(t_k)$

- **Problem:** learn periodic function $\Omega(\tau) : [0, T) \to \mathbb{R}^{n\times(n-1)}$

$$\zeta := \begin{bmatrix} \dot{x}_\perp \\ \dot{\tau} - 1 \end{bmatrix} = \Omega(\tau)\, x_\perp, \quad \Omega(\tau) = \begin{bmatrix} A(\tau) \\ g(\tau) \end{bmatrix}$$

- Assume smoothness of $\Omega(\tau)$ (requires smart choices of $S(\tau)$)
- **Method:** kernel-based identification

[1] I. R. Manchester, “Transverse dynamics and regions of stability for nonlinear hybrid limit cycles,” IFAC World Congress, 2011.

Kernel-based identification

- Many interpretations: **ridge** / kernel / Gaussian process regression ...
- **Basis decomposition:** high / infinite dimensional

$$\Omega_i(\tau) = \sum_{m=1}^{n_\psi} w_m^i \psi_m^i(\tau) = W_i \Psi_i(\tau), \quad n_\psi \to \infty$$

- **Regularized least squares:** ridge regularization

$$\min_{W_i} \quad \sum_{k=1}^{N} (\zeta_i(t_k) - W_i \Psi_i(\tau(t_k)) x_\perp(t_k))^2 + \lambda_i ||W_i||_2^2$$

- **Finite-dimensional solution:** linear w.r.t. kernel function evaluated at τ datapoints

$$\Omega_i(\tau) = \sum_{k=1}^{N} \alpha_{i,k}\, x_\perp(t_k)^\top K_i(\tau(t_k), \tau), \quad \underbrace{K_i(\tau, \tau')}_{\text{kernel function}} = \Psi_i(\tau)^\top \Psi_i(\tau')$$

Periodic kernel design

- Convert basis function design to kernel design
- ... but common kernels does not promote periodicity

- Periodic warping to obtain periodic kernels: $\chi(\tau) = \left[\sin(\frac{2\pi}{T}\tau)\ \cos(\frac{2\pi}{T}\tau)\right]^\top$
- Periodic square exponential kernel:

$$k^{\text{PSE}}(\tau, \tau') = \exp\left(-\frac{2\sin^2(\frac{\pi}{T^\star}(\tau - \tau'))}{l^2}\right)$$

- Hyperparameters estimated by maximum marginal likelihood (empirical Bayes)

Extensions

Additional **operating parameters** p

- Augment $\Omega(\tau)$ to $\Omega(\tau, p)$ with

$$k\left(\begin{bmatrix}\tau \\ p\end{bmatrix}, \begin{bmatrix}\tau' \\ p'\end{bmatrix}\right) = k^{\text{PSE}}(\tau, \tau') \cdot k^{\text{SE}}(p, p')$$

Exogenous inputs u

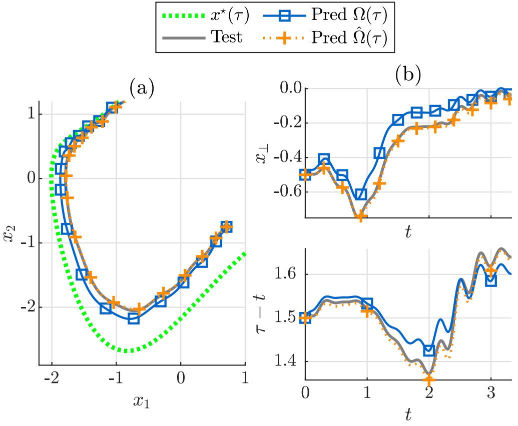
- Augment $x_\perp$ to $\begin{bmatrix}x_\perp \\ u\end{bmatrix}$

Example: Van der Pol oscillator

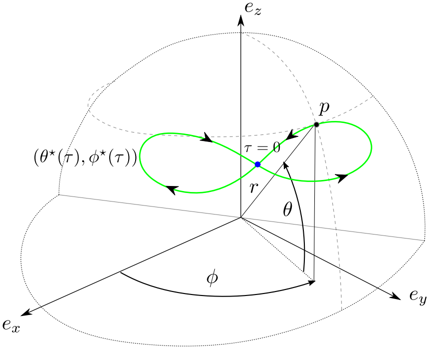
$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \mu(1 - x_1^2)x_2 - x_1 + \underbrace{D\sin(\omega t)}_{\text{exogenous input}}$$

$$\mu = D = 1, \ \omega = 10\omega^*$$

- Training data: 20 trajectories with 40 dB SNR
- $\Omega(\tau)$: analytical linearization
- $\hat{\Omega}(\tau)$: identified model



Application: airborne wind energy



- Power generation using tethered kites
- Unicycle kinematic model, figure-of-eight reference, periodically time-varying LQR controller[2]

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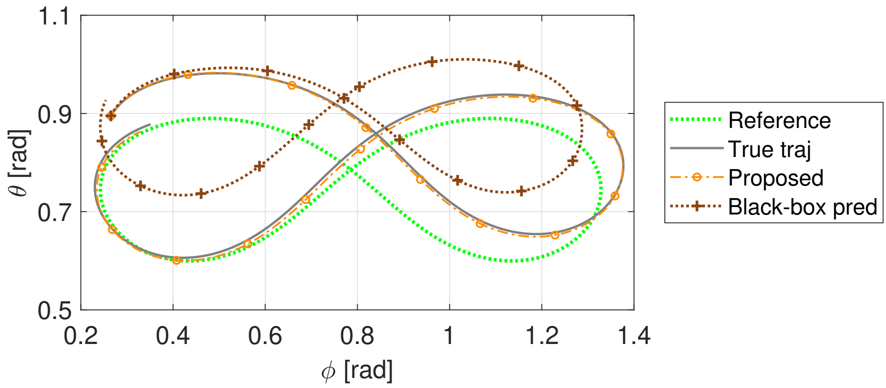
[2] E. Ahbe *et al.*, "Stability verification for periodic trajectories of autonomous kite power systems," European Control Conference, 2018.

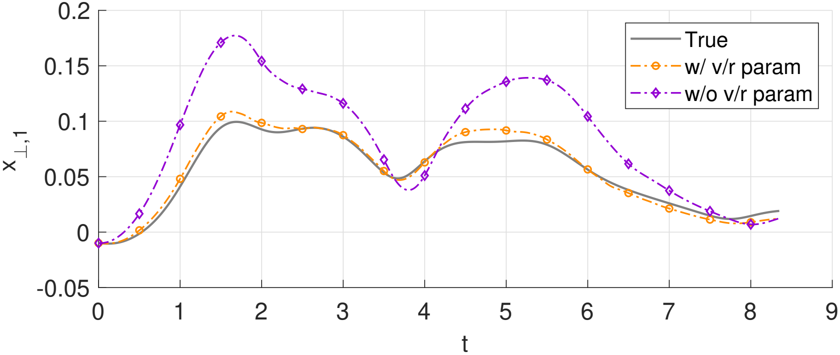
Application: airborne wind energy

- Training data: 16 loops from random initial conditions with 60 dB SNR



Application: airborne wind energy

- Additional operating parameter: $p = v/r$
- Training on 4 different p values



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Identify closed-loop dynamics with periodically parameter-varying models

- A grey-box approach using knowledge of the converging trajectory
- Identification as a periodic function learning problem, solved with kernel regression

- *Transversal hyperplane selection, discrete-time case, experimental application*

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