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ERNSI Workshop 2021

AUTOMATIC CONTROL LABORATORY

Signal Matrix Model in Simulation, Signal Denoising and Control Design

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Signal matrix model (SMM)

- **Challenges:**
  - systems are increasingly complex
  - how to use big data
- **Solution:** compact parametric models → implicit non-parametric trajectory models
- **Novelty:** a statistically optimal approach to deal with noisy data

- Construct trajectory by combining **direct knowledge** and **signal matrix**
- **Signal matrix:** Hankel matrix of trajectory data
- **Noise-free case:** *Willems' fundamental lemma*
- **Noisy case:** MAP estimation

$$\hat{\mathbf{z}} = \mathbf{z} + \mathbf{w}_\mathrm{z}$$

$$\mathbf{w}_\mathrm{z} \sim \mathcal{N}(0, \Sigma_\mathrm{z}), \mathbf{z} \sim \mathcal{N}\big(Zg, \Sigma_\mathrm{zg}(g)\big)$$

$\hat{\mathbf{z}}$: trajectory measurements
g: hyperparameters defining prior distribution

Applications

Simulation

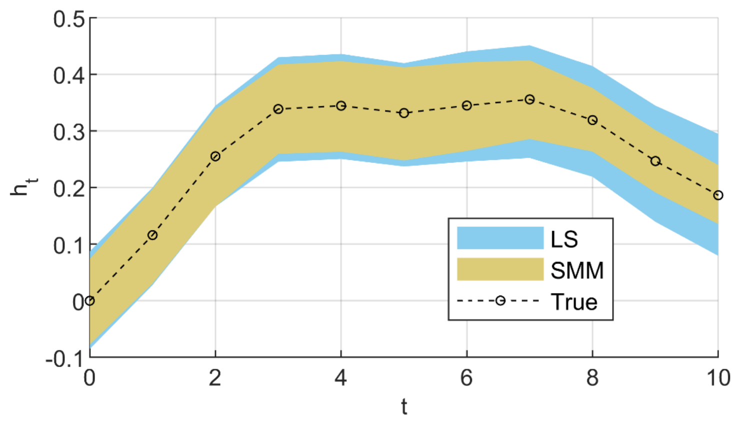
- Estimate outputs from known inputs and initial conditions
- **Condition:** $\mathbf{u}$ known exactly, first outputs $(\mathbf{y}_i)_{i=0}^{L_0-1}$ measured as initial condition
- Prior knowledge of $(\mathbf{y}_i)_{i=L_0}^{L-1}$ can be added as Gaussian process

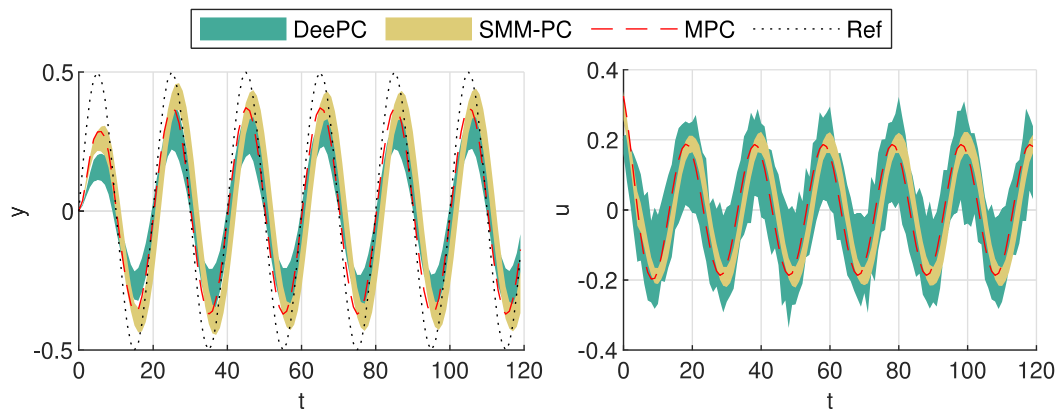
Signal denoising

- Denoise trajectory based on history trajectory data
- **Condition:** all the trajectories are measured with noise

Control design

- Optimal reference tracking by $\underset{\mathbf{u},\mathbf{y}}{\text{minimize}}\ \|\mathbf{y}-\mathbf{y}_\mathrm{ref}\|_Q^2 + \|\mathbf{u}-\mathbf{u}_\mathrm{ref}\|_R^2$
- **Condition:** $(\boldsymbol{u}_i, \mathbf{y}_i)_{i=0}^{L_0-1}$ measured past trajectory as initial condition
  $(\hat{\boldsymbol{u}}_i, \hat{\boldsymbol{y}}_i)_{i=L_0}^{L-1}$ set to reference trajectory
  corresponding elements in Σ_z are proportional to Q^{-1} & R^{-1}





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1 Signal matrix model (SMM)

Why?

Conventional **system identification paradigms** rely on compact parametric models.

Challenges: systems are increasingly complex;
how to use big data

Solution: moving from compact parametric models to implicit non-parametric trajectory models

Novelty: a statistically optimal approach to deal with noisy data

What?

Construct trajectory $\mathbf{z} = \mathrm{col}(\mathbf{u}, \mathbf{y})$ by combining **direct knowledge** and linear combination of **noise-corrupted signal matrix**.

Signal matrix: Hankel matrix of trajectory data

$$Z = \begin{bmatrix} z_0^d & z_1^d & \cdots & z_{M-1}^d \\ \vdots & \vdots & \ddots & \vdots \\ z_{L-1}^d & z_{L_0}^d & \cdots & z_{N-1}^d \end{bmatrix}$$

Preconditioning: compress by SVD

$$Z \xrightarrow{\text{svd}} WSV^{\mathsf{T}}, \qquad \tilde{Z} \triangleq WS(:, 1{:}Ln_z)$$

Noise-free case: *Willems' fundamental lemma* (Willems, 2005)

known part → $\mathbf{z}_1 = Z_1 g$, unknown part → $\mathbf{z}_2 = Z_2 g^*(\mathbf{z}_1, Z_1)$

Noisy case: $\hat{\mathbf{z}}$ as trajectory measurements; g as hyper-parameters defining prior distribution of $\mathbf{z}$ by Z

$$\hat{\mathbf{z}} = \mathbf{z} + \mathbf{w}_z, \qquad \mathbf{w}_z \sim \mathcal{N}(0, \Sigma_z), \qquad \mathbf{z} \sim \mathcal{N}(Zg, \Sigma_{zg}(g))$$

For unknown parts in $\hat{\mathbf{z}}$, corresponding elements in $\Sigma_z \to \infty$.

Empirical Bayes step: solve for g

$$\begin{aligned} g^\star &= \arg\max_g p(\hat{\mathbf{z}}|g) \\ &= \arg\min_g \operatorname{logdet}\left(\Sigma_{zg}(g) + \Sigma_z\right) + (\hat{\mathbf{z}} - Zg)^{\mathsf{T}}\left(\Sigma_{zg}(g) + \Sigma_z\right)^{-1}(\hat{\mathbf{z}} - Zg) \end{aligned}$$

MAP estimation step: solve for $\mathbf{z}$ given $g^\star$

$$\begin{aligned} \mathbf{z}^\star &= \arg\max_{\mathbf{z}} p(\hat{\mathbf{z}}|\mathbf{z}) \cdot p(\mathbf{z}) \\ &= \Sigma_{zg}(g^\star)\left(\Sigma_{zg}(g^\star) + \Sigma_z\right)^{-1}\hat{\mathbf{z}} + \Sigma_z\left(\Sigma_{zg}(g^\star) + \Sigma_z\right)^{-1} Zg^\star \end{aligned}$$

2 Applications

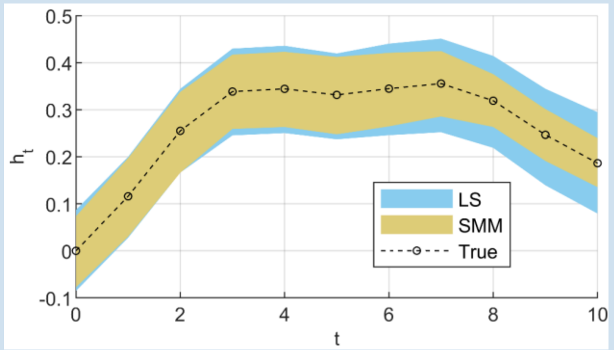
Simulation

Estimate outputs from known inputs and initial conditions.

Condition: $\mathbf{u}$ is known exactly, first outputs $(\mathbf{y}_i)_{i=0}^{L_0-1}$ are measured as initial condition

Prior knowledge of $(\mathbf{y}_i)_{i=L_0}^{L-1}$ can be added as Gaussian process.
e.g., stable spline kernels in impulse response simulation

Example: impulse response simulation **Benchmark:** least-squares estimation



Signal denoising

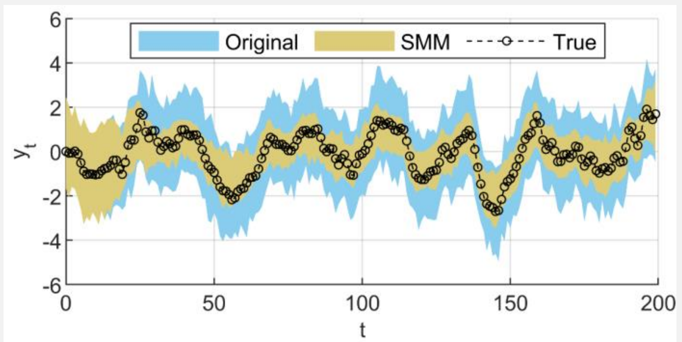
Denoise trajectory based on history trajectory data.

Condition: all the trajectories are measured with noise

Online data can be added to the signal matrix:

$$Z_{t+1} = [\gamma Z_t \quad (z_i)_{i=t-L+1}^{t}], \qquad \gamma: \text{forgetting factor}$$

Example: online signal denoising, Gaussian input



Control design

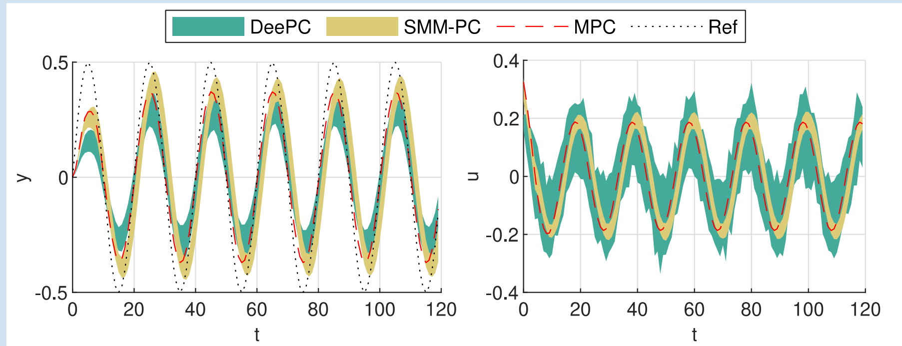
Optimal reference tracking by

$$\underset{\mathbf{u},\mathbf{y}}{\text{minimize}} \ \|\mathbf{y} - \mathbf{y}_{\text{ref}}\|_Q^2 + \|\mathbf{u} - \mathbf{u}_{\text{ref}}\|_R^2$$

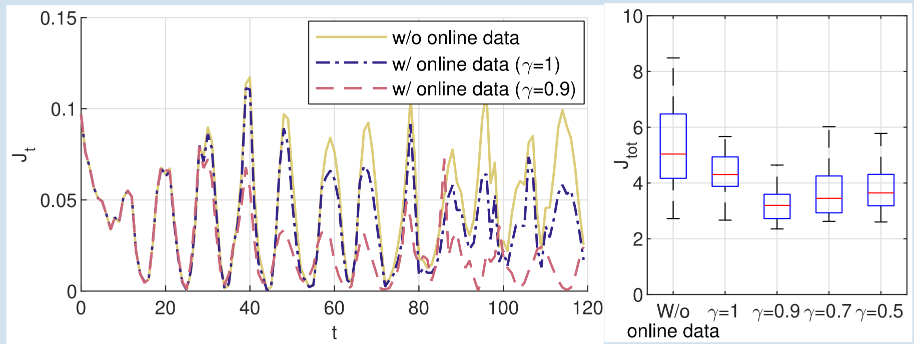
Condition: $(u_i, y_i)_{i=0}^{L_0-1}$ are measured past trajectory as initial condition; $(\hat{u}_i, \hat{y}_i)_{i=L_0}^{L-1}$ are set to reference trajectory, corresponding elements in Σ_z are proportional to Q^{-1} & R^{-1}.

Example: receding horizon, sinusoidal reference, no I/O constraints

Benchmark: ideal MPC & DeePC (*Coulson, 2019*)



Closed-loop trajectory comparison with ideal MPC & DeePC



Online data adaptation for system with slow parameter drifts

References

Mingzhou Yin, Andrea Iannelli, and Roy S. Smith. Maximum likelihood estimation in data-driven modeling and control. arXiv:2011.00925, 2020.

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