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AUTOMATIC CONTROL LABORATORY

Maximum Likelihood Signal Matrix Model for Data-Driven Predictive Control

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https://youtu.be/D02wY_DtoKM

1 Signal matrix model (SMM)

Why?

Model-based control requires accurate models of system response.

Challenges: systems are increasingly complex.

Solution: moving from compact parametric models to non-parametric input-output mapping.

Novelty: a statistically optimal approach to deal with noisy data.

What?

An **implicit input-output mapping** derived directly from the signal matrix of **noise-corrupted** data.

Signal matrix: Hankel matrix of input-output trajectory data

$$U=\begin{bmatrix} u_0^d & u_1^d & \cdots & u_{M-1}^d \\ \vdots & \vdots & \ddots & \vdots \\ u_{L-1}^d & u_{L_0}^d & \cdots & u_{N-1}^d \end{bmatrix},\qquad Y=\begin{bmatrix} y_0^d & y_1^d & \cdots & y_{M-1}^d \\ \vdots & \vdots & \ddots & \vdots \\ y_{L-1}^d & y_{L_0}^d & \cdots & y_{N-1}^d \end{bmatrix}$$

Noise-free case: *Willems' fundamental lemma* (Willems, 2005)

$$\begin{bmatrix}\mathbf{u}_{\text{ini}}\\ \mathbf{y}_{\text{ini}}\\ \mathbf{u}\end{bmatrix}=\begin{bmatrix}U_p\\ Y_p\\ U_f\end{bmatrix}g,\qquad \mathbf{y}=Y_f\, g^\star(\mathbf{u},\mathbf{u}_{\text{ini}},\mathbf{y}_{\text{ini}};U_f,U_p,Y_p)$$

Noisy case: *How to estimate g?* Maximum likelihood estimation

$$g^\star=\arg\max_g p(\varepsilon_y,\mathbf{y}\mid g)$$
$$=\arg\min_{g\in\mathcal{G}} \log\det\Sigma_y(g)+\begin{bmatrix}Y_p g-\mathbf{y}_{\text{ini}}\\ 0\end{bmatrix}^{\mathrm{T}}\Sigma_y^{-1}(g)\begin{bmatrix}Y_p g-\mathbf{y}_{\text{ini}}\\ 0\end{bmatrix}$$

posterior covariance

How?

An approximate iterative QP algorithm:

$$g^{k+1}=\arg\min_{g\in\mathcal{G}} \lambda(g^k)\|g\|_2^2+\|Y_p g-\mathbf{y}_{\text{ini}}\|_2^2,\qquad \lambda(g^k)=\frac{L'\sigma_p^2}{\|g_k\|_2^2}+L\sigma^2$$

Closed-form solution:

$$g^{k+1}=\mathcal{P}(g^k)\,\mathbf{y}_{\text{ini}}+\mathcal{Q}(g^k)\begin{bmatrix}\mathbf{u}_{\text{ini}}\\ \mathbf{u}\end{bmatrix}$$

2 Data-Driven Predictive Control with SMM

Idea

Optimal reference tracking by receding horizon control:

$$\underset{\mathbf{u},\mathbf{y}}{\text{minimize}}\ \sum_{k=0}^{L'-1}\left(\|y_k-r_{t+k}\|_Q^2+\|u_k\|_R^2\right)$$
$$\text{subject to}\ \left(\begin{bmatrix}\mathbf{u}_{\text{ini}}\\ \mathbf{u}\end{bmatrix},\begin{bmatrix}\mathbf{y}_{\text{ini}}\\ \mathbf{y}\end{bmatrix}\right)\ \text{is a possible system trajectory}\ (*)$$
$$\mathbf{u}\in\mathcal{U},\qquad \mathbf{y}\in\mathcal{Y}$$

Instead of a compact model, $(*)$ can be formulated in a data-driven approach (Coulson, 2019).

With SMM: $\mathbf{y}=Y_f\, g^*_{SMM}(\mathbf{u},\mathbf{u}_{\text{ini}},\mathbf{y}_{\text{ini}})$

Computation

1) Approximate the signal matrix model with one iteration, warm-starting from the previous g-value → linear constraint

$$(*)\ \leftrightarrow\ \mathbf{y}=Y_f\left(\mathcal{P}(g^{t-1})\,\mathbf{y}_{\text{ini}}+\mathcal{Q}(g^{t-1})\begin{bmatrix}\mathbf{u}_{\text{ini}}\\ \mathbf{u}\end{bmatrix}\right)$$

2) Precondition by **compressing the signal matrix** → online computations does not depend on data length

$$\begin{bmatrix}U\\ Y\end{bmatrix}\overset{\textbf{svd}}{\longrightarrow}WSV^{\mathrm{T}},\qquad \begin{bmatrix}\tilde{U}\\ \tilde{Y}\end{bmatrix}\triangleq WS(:,1:2L)$$

Salient features

1) **Incorporation of online data:** online measurements added to the signal matrix

forgetting factor

$$\begin{bmatrix}U_{t+1}\\ Y_{t+1}\end{bmatrix}=\begin{bmatrix}\gamma U_t & (u_i)_{i=t-L+1}^t\\ \gamma Y_t & (y_i)_{i=t-L+1}^t\end{bmatrix}$$

Beneficial for low SNR data and/or slowly varying systems

2) **Regularized SMM predictive control:** additional objective of reducing prediction error

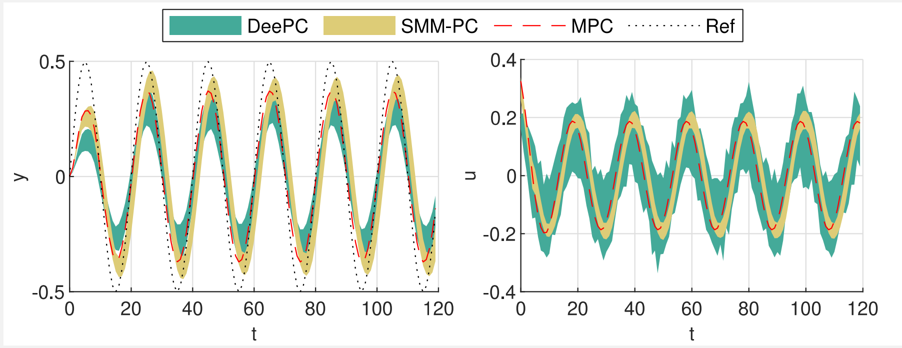
$$\left(\Sigma_{y_f}\right)_{i,i}=\sigma^2\|g\|_2^2,\qquad J_{\text{Reg-SMM}}=J_{\text{ctr}}+\zeta\cdot\sigma^2\|g^t\|_2^2$$

Tune ζ to add robustness to the controller

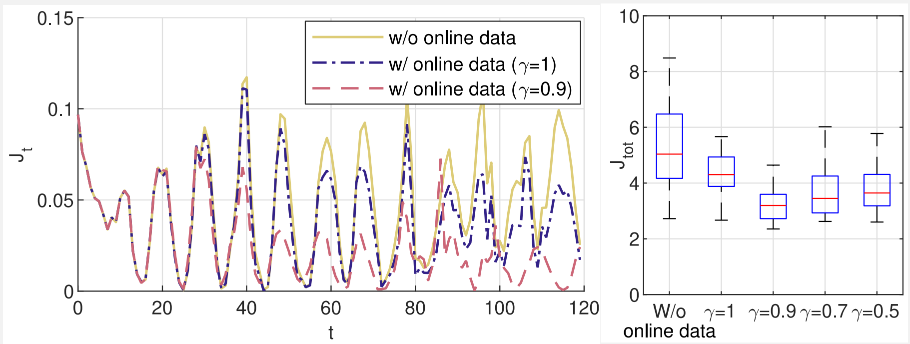
3 Results

Case study: 4th-order linear system, sinusoidal reference, no I/O constraints, known noise levels.

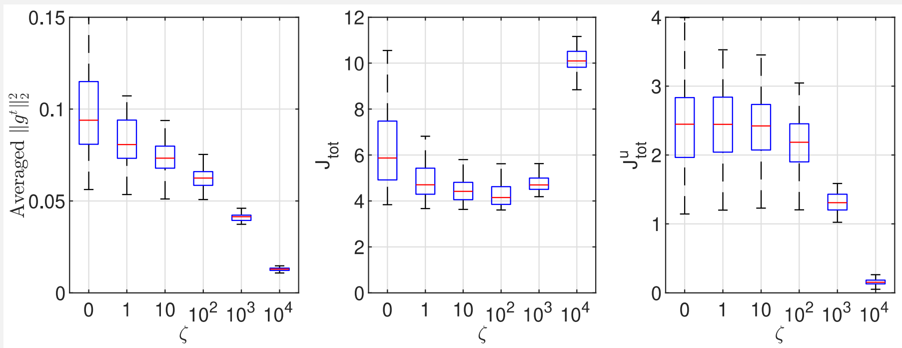
Benchmark: ideal MPC & DeePC (*Coulson, 2019*)



Closed-loop trajectory comparison with ideal MPC & DeePC



Online data adaptation for system with slow parameter drifts



Performance of regularized SMM predictive control

References

Mingzhou Yin, Andrea Iannelli, and Roy S. Smith. Maximum likelihood estimation in data-driven modeling and control. arXiv preprint arXiv:2011.00925, 2020.